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Using Dynamic Analysis for Compact Gear Design

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ABSTRACT

This paper presents procedures for designing compact spur gear sets with the objective of minimizing the gear size. The allowable tooth stress and dynamic response are incorporated in the process to obtain a feasible design region. Various dynamic rating factors were investigated and evaluated. The constraints of contact stress limits and involute interference combined with the tooth bending strength provide the main criteria for this investigation. A three-dimensional design space involving the gear size, diametral pitch, and operating speed was developed to illustrate the optimal design of spur gear pairs.

The study performed here indicates that as gears operate over a range of speeds, variations in the dynamic response change the required gear size in a trend that parallels the dynamic factor. The dynamic factors are strongly affected by the system natural frequencies. The peak values of the dynamic factor within the operating speed range significantly influence the optimal gear designs. The refined dynamic factor introduced in this study yields more compact designs than AGMA dynamic factors.

INTRODUCTION

Designing compact (minimum size) gear sets provides benefits such as minimal weight, lower material cost, smaller housings, and smaller inertial loads. Gear designs must satisfy constraints, including bending strength limits, pitting resistance, and scoring. Many approaches for improved gear design have been proposed in previous literature (Refs. 1 to 14). Among those, the use of optimization techniques has received much attention (Refs. 9 to 13). However, these studies dealt primarily with static tooth strength. Dynamic effects must also be considered in designing compact gear sets.

Previous research presented different approaches for optimal gear design. Reference 9 considered involute interference, contact stresses, and bending fatigue. They concluded that the optimal design usually occurs at the intersection point of curves relating the tooth numbers and diametral pitch required to avoid pitting and scoring. Reference 10 expanded the model to include the AGMA geometry factor and AGMA dynamic factor in the tooth strength formulas. Their analysis found that the theoretical optimal gear set occurred at the intersection of the bending stress and contact stress constraints at the initial point of contact.

More recently, the optimal design of gear sets has been expanded to include a wider range of considerations. Reference 11 approached the optimal strength design for nonstandard gears by calculating the hob offsets to equalize the maximum bending stress and contact stress between the pinion and gear. Reference 12 treated the entire transmission as a complete system. In addition to the gear mesh parameters, the selection of bearing and shaft proportions were included in the design configuration. The mathematical formulation and an algorithm are introduced in (Ref. 13) to solve the multiobjective gear design problem, where feasible solutions can be found in a three-dimensional solution space.

Most of the foregoing literature dealt primarily with static tooth strength. These studies use the Lewis formula assuming that the static load is applied at the tip of the tooth. Some considered stress concentration and the AGMA geometry and dynamic factors. However, the operating speed must be considered for dynamic effects. Rather than using the AGMA dynamic factor, which increases as a simple function of pitch line velocity; the gear dynamics code DANST (Dynamic ANalysis of Spur gear Transmissions) (Refs. 1 to 3) was used here to calculate a dynamic load factor.

The purpose of the present work is to develop a procedure to design compact spur gear sets including dynamic considerations. Since root fillet stress is important in determining tooth-bending failure in gear transmission, the modified Heywood (Refs. 14 and 15) formula is used. Constraint criteria employed for this investigation include the involute interference limits combined with the tooth bending strength and contact stress limits. This study was limited to spur gears with standard involute tooth profile.

MODEL FORMULATION

Objective Function

The design objective of this study is to obtain the most compact gear set satisfying design requirements that include loads and power level, gear ratio and material parameters. The gears designed must satisfy operational constraints including avoiding interference, pitting, scoring distress and tooth breakage. The required gear center distance C is the chosen parameter to be optimized.

$$C = R_{p1} + R_{p2} \qquad (1)$$

where

R_{p1} pitch radius of gear 1
R_{p2} pitch radius of gear 2

Design Parameters and Variables

The following table lists the parameters and variables used in this study:

Table 1.—Basic Gear Design Parameters and Variables

Gear parameters	Design variables
Bending and contact strenght limits	Number of pinion teeth
Operating torque	Diametral pitch
Gear speed ratio	Operating speed
Face width	
Pressure angle	

Design Constraints

Involute Interference. Involute interference is defined as a condition in which there is an obstruction on the tooth surface that prevents proper tooth contact (Ref. 17); or contact between portions of tooth profiles that are not conjugate (Ref. 18). Interference occurs when the driven gear contacts a noninvolute portion (below the base circle) of the driving gear. Undercutting occurs during tooth generation if the cutting tool removes the interference portion of the gear being cut. An undercut tooth is weaker, less resistant to bending stress, and prone to premature tooth failure. DANST has a built-in routine to check for interference.

Bending Stress. Tooth bending failure at the root is a major concern in gear design. If the bending stress exceeds the fatigue strength, the gear tooth has a high probability of failure. The AGMA bending stress equation can be found in Ref. 10 and also in other gear literature. In this study, a modified Heywood formula for tooth root stress was used to compare with the AGMA equation. This formula correlates well with experimental data and finite element analysis results (Ref. 14):

$$\sigma_j = \frac{W_j \cos\beta_j}{F}\left[1+0.26\left(\frac{h_f}{2R_f}\right)^{0.7}\right]\left[\frac{6l_f}{h_f^2}+\sqrt{\frac{0.72}{h_f l_f}}\left(1-\frac{h_L}{h_f}\nu\tan\beta_j\right)-\frac{\tan\beta_j}{h_f}\right] \qquad (2)$$

where

σ_j root bending stress at loading position j.
W_j transmitted load at loading position j.
β_j load angle, degree
F face width of gear tooth, inch
n approximately 1/4, according to Heywood (Ref. 15).
R_f fillet radius, inch

other nomenclature is defined in Fig. 1 and Refs. 14 and 15. To avoid tooth failure, the bending stress should be limited to the allowable bending strength of the material as suggested by AGMA (Ref. 19),

$$\sigma_j \le \sigma_{all} = \frac{S_t K_L}{K_T K_R K_v} \qquad (3)$$

where

σ_{all} allowable bending stress
S_t AGMA bending strength
K_L life factor
K_T temperature factor
K_R reliability factor
Kv dynamic factor

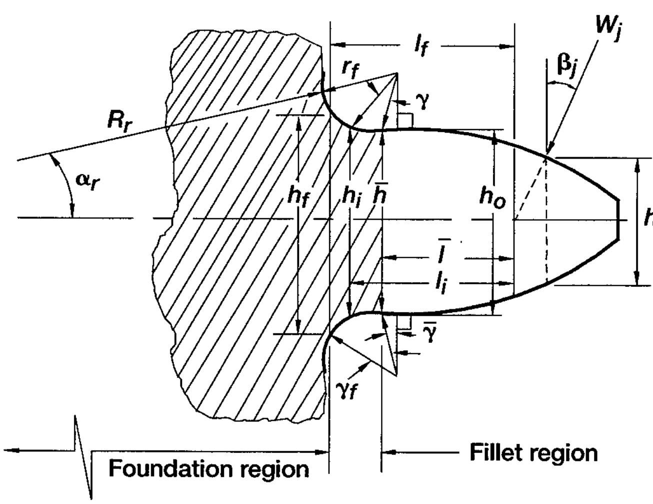

Figure 1.—Tooth geometry nomenclature for root stress calculation [14].

Surface Stress. The surface failure of gear teeth is an important concern in gear design. Surface failure modes include pitting, scoring and wear. Pitting is a gear tooth surface failure caused by the formation of cavities on the tooth surface as a result of repeated stress applications. Scoring is another surface failure that usually results from high loads or lubrication problems. It is defined as the rapid removal of metal from a tooth surface caused by the tearing out of small particles that have welded together as a result of metal-to-metal contact. The surface is characterized by a ragged appearance with furrows in the direction of tooth sliding (Ref. 20). Wear is a fairly uniform removal of material from the tooth surface.

The stresses on the surface of gear teeth are determined by formulas derived from the work of Hertz (Ref. 17). The Hertzian contact stress between meshing teeth can be expressed as

$$\sigma_{Hj} = \sqrt{\frac{W_j \cos\beta_j}{\pi F \cos\phi}\left[\frac{\frac{1}{\rho_1}+\frac{1}{\rho_2}}{\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}}\right]} \tag{4}$$

where

σ_{Hj} contact stress at loading position j
W_j transmitted load at loading position j.
β_j load angle, degree
F face width of gear tooth, inch
ϕ pressure angle, degree
$\rho_{1,2}$ radius of curvature of gear 1,2 at the point of contact, inch
$n_{1,2}$ Poisson's ratio of gear 1,2
$E_{1,2}$ modulus of elasticity of gear 1,2, psi

The AGMA recommends that this contact stress should also be considered in a similar manner as the bending endurance limit (Ref. 19). The equation is

$$\sigma_{Hj} \le \sigma_{c,all} = S_c \frac{C_L C_H}{C_T C_R} \tag{5}$$

where

$\sigma_{c,all}$ allowable contact stress
S_c AGMA surface fatigue strength
C_L life factor
C_H hardness-ratio factor
C_T temperature factor
C_R reliability factor

According to Savage et al. (Ref. 9), Hertzian stress is a measure of the tendency of the tooth surface to develop pits and is evaluated at the lowest point of single tooth contact rather than at the less critical pitch point as recommended by AGMA. Gear tip scoring failure is highly temperature dependent (Ref. 20) and the temperature rise is a direct result of the Hertz contact stress and relative sliding speed at the gear tip. Therefore, the possibility of scoring failure can be determined by Eq. (4) with the contact stress evaluated at the initial point of contact. A more rigorous method not used here is to use the PVT equation or the Blok scoring equation. (See Ref. 17).

Dynamic Load Effect. One of the major goals of this work is to study the effect of dynamic load on optimal gear design. The dynamic load calculation is based on the NASA gear dynamics code DANST. DANST has been validated with experimental data for high-accuracy gears at NASA Lewis Research Center (Ref. 21). DANST considers the influence of gear mass, meshing stiffness, tooth profile modification, and system natural frequencies in its dynamic calculations.

The dynamic tooth load depends on the value of relative dynamic position and backlash of meshing tooth pairs. After the gear dynamic load is found, the dynamic load factor can be determined by the ratio of the maximum gear dynamic load during mesh to the applied load. The applied load equals the torque divided by the base circle radius. This ratio indicates the relative instantaneous gear tooth load. Compact gears designed using the dynamic load calculated by DANST will be compared with gears designed using the AGMA suggested dynamic factor, which is a simple function of the pitch line velocity.

GEAR DESIGN APPLICATION

Design Algorithm

An algorithm was developed to perform the analyses and find the optimum gear design. The process starts with the input of gear parameters such as geometry, applied load, speed, diametral pitch, pressure angle, and tooth numbers.

For this study, the diametral pitch was varied from two to twenty. Static analysis was performed to check for involute interference and to calculate the meshing stiffness variations and static transmission errors of the gear pair. If there was a possibility of interference, the number of pinion teeth was increased by one and the static process was repeated. Results from the static analyses were incorporated in the equations of motion of the gear set to obtain the dynamic motions of the system. Instantaneous dynamic load at each contact point along the tooth profile was determined from these motions. The contact stress and root bending stresses were calculated from the dynamic response.

If all the calculated stresses are less than the design stress limits for a possible gear set, the data for this set were added to a candidate group. At each value of diametral pitch, the most compact gear set in the candidate group will have the smallest center distance. These different candidate designs can be compared in a table or graph to show the optimum design from all the sets studied.

The analyses above are for gears operating at a single speed (in this case, 1120 rpm input speed). To examine the effect of varying speed, the analyses can be repeated at different speeds. As the speed varies, the optimal gear sets determined for each speed can be collected to form a design space. The study to follow presents a three-dimensional design space to find the minimum center distance as a function of rotation speed, pinion tooth number, and diametral pitch.

Design Example

Table 2 shows the basic gear parameters for a sample gear set to be studied. They were first used in a gear design problem by Shigley and Mitchell (Ref. 18), and later used by Carroll and Johnson (Ref. 10) as an example for optimal design of compact gear sets. The sample gear set transmits 100 horsepower at an input speed of 1120 rpm. The gear set has standard full depth teeth and a speed reduction ratio of 4. In this study, the face width of the gear is always chosen to be one-half the pinion pitch diameter. In other words, the length to diameter ratio λ is 0.5.

In Carroll's study, the AGMA dynamic factor chosen represents medium to low accuracy gears with teeth finished by hobbing or shaping (Ref. 19). The dynamic factor formula is given by:

Table 2.—Basic Design Parameters of Sample Gear Set

Pressure angle, ϕ, degrees	20
Gear ratio, M_g	4.0
Length to diameter ratio, λ	0.5
Transmitted power, hp	100
Applied torque, lb-in.	5627.264
Input speed, rpm	1120
Modulus of Elasticity, E, psi	30×10^6
Poisson's ratio, ν	0.3
Scoring and pitting stress limits, S_S and S_P, psi	79 230
Bending stress limit, S_b, psi	19 810

Table 3.—Carroll's optimization results of sample gear set (Ref. 10) (Using Lewis tooth stress formula)

Pd	NT1	NT2	CD	FW	CR	Sb	Ss	Sp
2.00	19	76	23.750	4.750	1.681	2.872	72.497	51.396
2.25	20	80	22.222	4.444	1.691	3.553	72.162	55.853
2.50	21	84	21.000	4.200	1.701	4.275	72.532	59.950
3.00	23	92	19.167	3.833	1.717	5.820	74.100	67.213
4.00	27	108	16.875	3.375	1.745	9.202	77.876	78.860
6.00	40	160	16.667	3.333	1.805	12.703	66.972	78.309
8.00	53	212	16.563	3.313	1.840	16.191	63.302	78.247
10.00	**66**	**264**	**16.500**	**3.300**	**1.863**	**19.670**	**61.463**	**78.285** ←
12.00	86	344	17.917	3.583	1.887	19.727	53.219	69.603
16.00	132	528	20.625	4.125	1.917	19.726	42.459	57.137
20.00	185	740	23.125	4.625	1.934	19.742	35.708	48.760

$$K_v = \frac{50}{50+\sqrt{V}} \tag{6}$$

where V is the pitch line velocity in feet per minute. Since K_v appears in the denominator of the AGMA root stress equation, the root stress calculated at high speeds rises as the one-half power of the speed

Table 3 displays Carroll's (Ref. 10) optimal design results for the sample gears. The optimal design is indicated in bold type and by an arrow. In the table, P_d is the diametral pitch, NT1 and NT2 represent the number of teeth of pinion and gear, respectively, CD is the center distance, FW is tooth face width, CR is contact ratio, S_b, S_s, and S_p are the calculated maximum values for bending, scoring and pitting stress, respectively. The theoretical optimum for this example occurs at the intersection of bending stress and contact stress constraint curves at the lowest point of single tooth contact. This creates a gear set that has NT1 $\cong$ 64 and $P_d \cong 9.8$ for a theoretical center distance of 16.333 in. The minimum practical center distance (16.50 in.) is obtained when NT1 = 66 and $P_d = 10.0$

For comparison with the above results, we used the same AGMA dynamic factor K_v (Eq. 6) but with the modified Heywood tooth bending stress formula (Eq. 2) in the calculations. Table 4 lists the optimization results obtained. As can be seen from the table, the minimum practical center distance (16.750 in.) is obtained when NT1 = 67 and $P_d = 10.0$. This is very close to Carroll's design but his optimal gear set will exceed the design limit of 19.81 Kpsi (from Table 2) for maximum bending stress on the pinion according to our calculations. The differences between Carroll's results and those reported here are likely due to the use of different formulas for bending stress calculations.

Figure 2 shows graphically the design space for the results presented in Table 4, depicting the stress constraint curves of bending, scoring, and pitting. The region above each constraint curve indicates feasible design space for that particular constraint. In the figure, the theoretical optimum is located at the intersection point of the scoring stress and the bending stress constraint.

Table 5 shows the optimization results for the design example using the dynamic analysis program DANST which calculates the instantaneous dynamic tooth load at each gear contact position by solving the equations of motion. This instantaneous tooth load is then used to determine tooth bending stress using the modified Heywood formula. DANST assumes high quality gears. Dynamic load effects determined from DANST will be lower than that from the AGMA formula used in this study. Therefore, using DANST to calculate the dynamic factor may lead to more compact optimum gears than using the AGMA dynamic factor.

From Table 5, we can see that the optimal gear set has a smaller center distance than those found earlier. The optimum gear set using the DANST dynamic model has a center distance of 13.75 in. with NT1 = 33 and $P_d = 6.0$. In other words, a more compact design was found. Note that a design with the minimum number of pinion teeth is not necessarily the smallest gear set since the size of the teeth (as given by the diametral pitch) also affects the center distance. This can be better illustrated in Fig. 3. Figure 3(a) shows a feasible design space bounded by a constraint curve that relates the minimum number of teeth on the pinion to the

Table 4.—Optimization results of sample gear
(Using modified Heywood tooth stress formula)

Pd	NT1	NT2	CD	FW	CR	Sb	Ss	Sp
2.00	19	76	23.750	4.750	1.681	2.847	66.873	52.390
2.25	19	76	21.111	4.222	1.681	3.953	78.539	61.589
2.50	20	80	20.000	4.000	1.691	4.718	76.864	65.812
3.00	22	88	18.333	3.667	1.709	6.384	76.054	73.234
4.00	28	112	17.500	3.500	1.751	8.466	66.656	76.216
6.00	41	164	17.083	3.417	1.808	12.215	58.956	77.066
8.00	54	216	16.875	3.375	1.842	15.785	56.336	77.689
10.00	**67**	**268**	**16.750**	**3.350**	**1.865**	**19.369**	**55.015**	**78.137**
12.00	87	348	18.125	3.625	1.888	19.637	47.915	69.817
16.00	134	526	20.938	4.188	1.918	19.638	38.076	57.045
20.00	188	752	23.500	4.700	1.935	19.718	32.006	48.616

Table 5.—Optimization results of sample gear—DANST
(Using refined K_v and modified Heywood stress formula)

Pd	NT1	NT2	CD	FW	CR	Sb	Ss	Sp
2.00	19	76	23.750	4.750	1.681	1.920	63.565	28.973
2.25	20	80	22.222	4.444	1.691	2.427	63.811	31.721
2.50	20	80	20.000	4.000	1.691	3.178	79.176	39.602
3.00	22	88	18.333	3.667	1.709	4.402	76.971	44.148
4.00	26	104	16.250	3.250	1.739	7.156	71.988	51.447
6.00	**33**	**132**	**13.750**	**2.750**	**1.777**	**14.613**	**74.479**	**63.930**
8.00	50	200	15.625	3.125	1.833	15.017	75.324	47.289
10.00	59	236	14.750	2.950	1.852	16.907	72.216	52.703
12.00	77	308	16.042	3.208	1.878	16.165	59.231	45.701
16.00	99	396	15.469	3.094	1.898	17.297	50.546	51.125
20.00	143	572	17.875	3.575	1.921	17.596	38.923	39.042
24.00	172	688	17.917	3.583	1.931	18.496	35.306	37.697

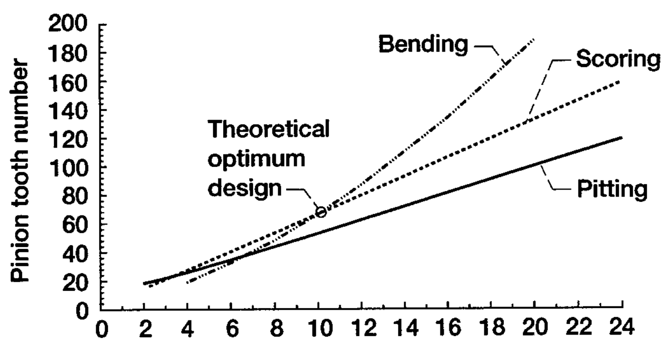

Figure 2.—Design space of stress constraints for sample gears.

diametral pitch. This can be converted into another design space, in terms of center distance, as shown in Fig. 3(b). In this figure the point that corresponds to the lowest center distance on the feasible design curve indicates the most compact design. This design has a diametral pitch of 6, therefore, from Fig. 3(a) it must have at least 33 teeth.

Compact Gears Designed for a Range of Speeds

The foregoing examples considered only a single input speed of 1120 rpm. The dynamic response of gear sets can be significantly affected by different operating speeds. The effect of varying speed on optimal compact gear design will be investigated below.

Using the parameters of the sample gears, we consider speeds from 1,120 to 11,120 rpm, with an increment of 500 rpm. Figure 4(a) displays the curves showing the optimum pinion tooth number as a function of operating speed at different diametral pitch values. The diametral pitch was varied from 2.0 to 24.0. The curves show little variation with speed. This indicates that the optimum tooth number changes little with speed. The peak value of each curve shows where a larger gear was required due to dynamic effects. This phenomenon is similar to that of the dynamic factor curve in the gear literature (Ref. 19).

The minimum tooth numbers, obtained from Fig. 4(a), indicates the most compact gear design at each diametral pitch if the input speed is fixed. However, an optimal compact gear set (with overall minimum center distance) cannot be determined from this figure. A gear set with the minimum number or teeth is not necessarily the most compact configuration because the center distance also depends upon the diametral pitch. The data in Fig. 4(a) can be converted to a more useful form, Fig. 4(b), to illustrate directly the relation between speed and minimum gear size.

Each curve in Fig. 4(b) depicts the relationship between the center distance and input speed for one specific diametral pitch. Using both Figs. 4(a) and (b) as design aids, we can determine the most compact gear set not only at a single operating speed but also over a desired range of speeds. For example, at the single speed of 1120 rpm, the most compact design can be found starting in Fig. 4(b) by locating the lowest point (curve) of all curves at this speed. In this case, the optimal compact gear set has $P_d = 6.0$ and a center distance of 13.75 in. Then we find in Fig. 4(a), the number of pinion teeth required for this optimal gear set is NT1 = 33. This is the same as the design result displayed in Table 5.

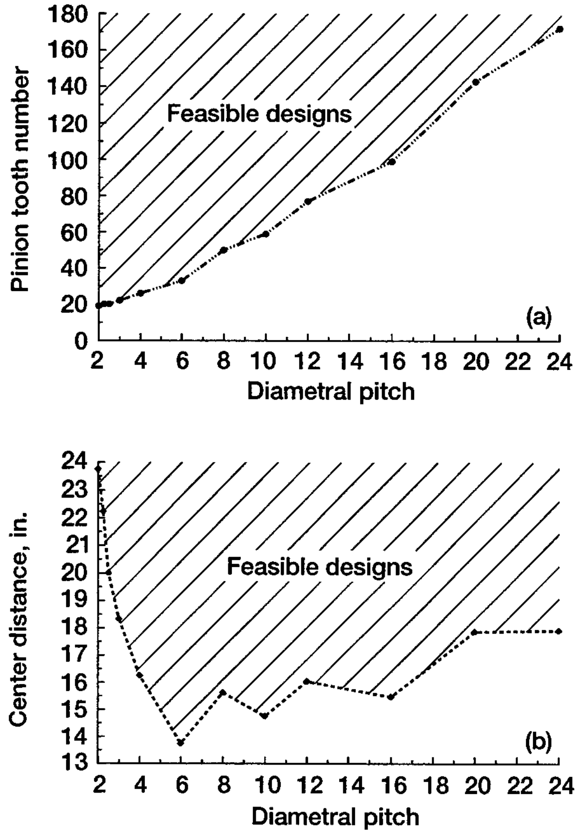

Figure 3.—Design space to determine optimal gear set of sample gears at an input speed of 1120 rpm. (a) Required number of pinion teeth versus diametral pitch. (b) Center distance versus diametral pitch.

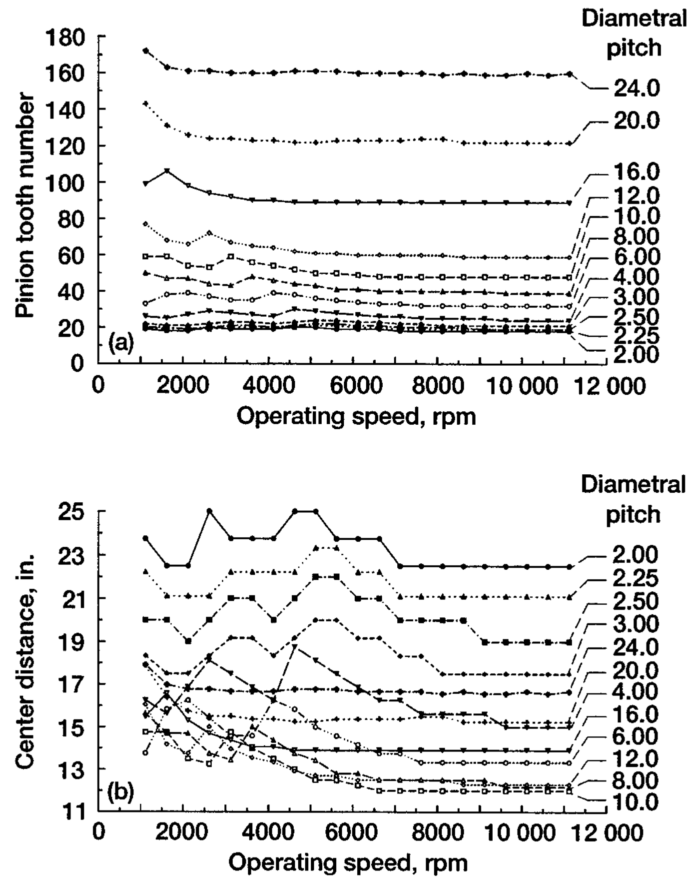

Figure 4.—Effects of speed on pinion tooth number and center distance of optimal gear sets using DANST for dynamic analysis. (a) Required number of pinion teeth versus speed. (b) Center distance versus speed.

To design a compact gear set for operation over a range of speeds, we can compare the curves in Fig. 4(b) and select the one with the overall smallest peak value within the speed range. For example, if the desired operating speeds are between 3000 and 5000 rpm, it can be seen from Figs. 4(b) and (a) that the optimal compact gear set should have $P_d = 12.0$, NT1 = 68, and a center distance of 14.167 in. This design satisfies all stress constraints under both static and dynamic considerations. For high-speed gears to be operated mostly at greater than 5000 rpm, a gear set with $P_d = 10.0$ appears to be the best choice for the optimal compact design.

To better visualize the design procedure, a three-dimensional design space, Fig. 5(a), was developed by incorporating the diametral pitch as an additional parameter into Fig. 4(b). This figure eliminates the clutter due to curve overlap in Fig. 4(b). From this figure, we can more easily identify the region of the most compact gear sets for any speed and diametral pitch. Gear sets with a diametral pitch of 10.0 may offer the best design because they appear to have the lowest center distance values. The design space of Fig. 5(a) can also be used to evaluate a gear set designed by other means. If the gear set is located on or above the design surface, the design is adequate and satisfies all the stress constraints, otherwise the gear set should not be used.

Figure 5(b) displays the effects of diametral pitch and operating speed on gear center distance as a contour diagram. For the speed range considered in the study, the most compact gear sets have a diametral pitch between 8.0 to 12.0. If the diametral pitch is less than 6.0, the required center distance increases significantly regardless of the operating speed. This figure may complement Fig. 5(a) as a tool for developing compact gear sets.

The design curves shown in Figs. 4 and 5 are valid only for the basic gear parameters of the sample gears shown in Table 2. Different basic parameters will require new design curves. However, the design procedures remain the same and are applicable to all standard and nonstandard spur gears with involute tooth profile.

CONCLUSIONS

This paper presents a method for optimal design of standard spur gears for minimum dynamic response. A study was performed using a sample gear set from the gear literature. Optimal gear sets were compared for designs based on the AGMA dynamic factor and a refined dynamic factor calculated using the DANST gear dynamics code. A three-dimensional design space for designing optimal compact gear sets

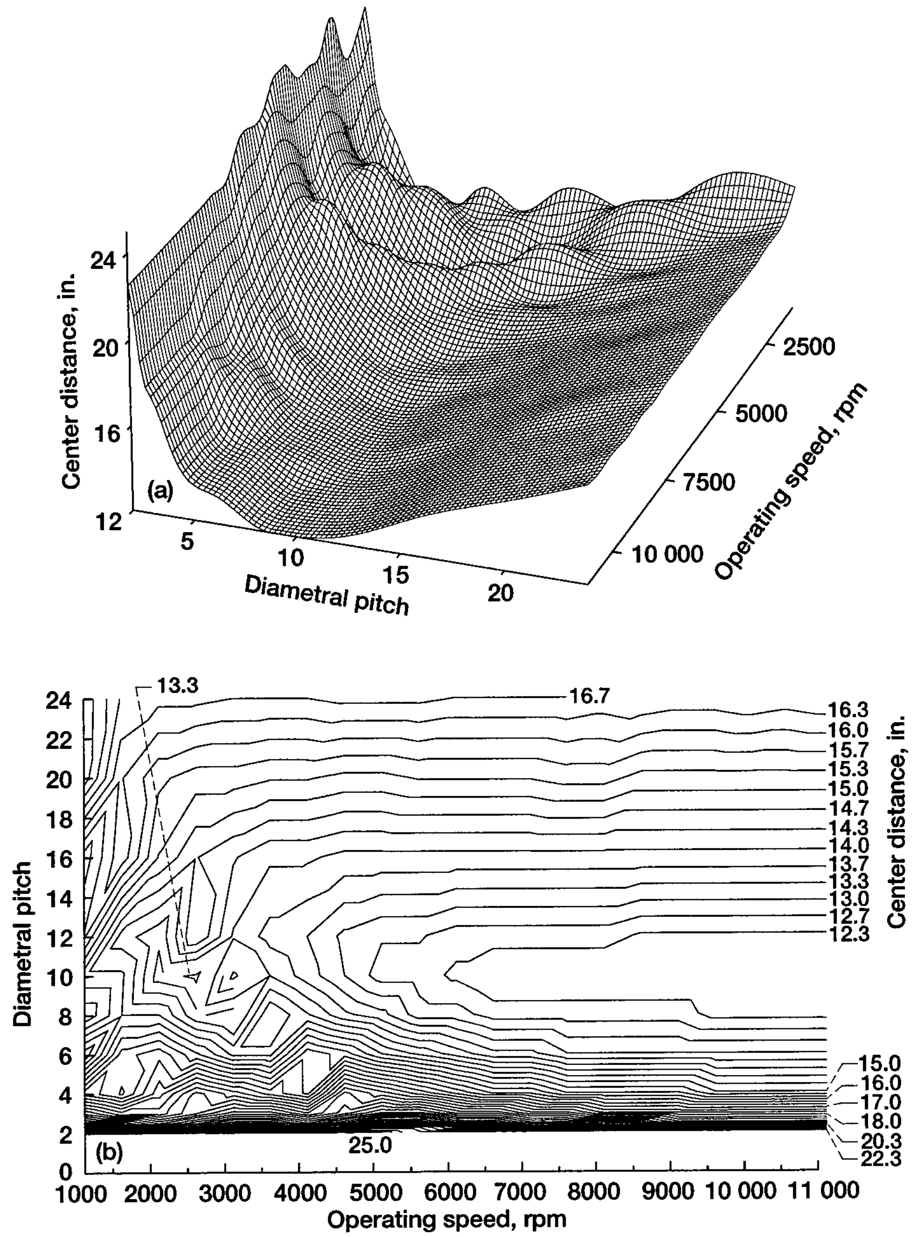

Figure 5.—Three-dimensional design space and contour diagram of sample gears using DANST for dynamic analysis. (a) Three-dimensional design space. (b) Contour plot.

was developed. The operating speed was varied over a broad range to evaluate its effect on the required gear size. The following conclusions were obtained:

1. The required size of an optimal gear set is significantly influenced by the dynamic factor. The peak dynamic factor at system natural frequencies dominates the design of optimal gear sets that operate over a wide range of speeds.

2. A refined dynamic factor calculated by the dynamic gear code DANST allows a more compact gear design than the AGMA dynamic factors. This is due to the more realistic model as well as the higher quality gears assumed by DANST.

3. Compact gears designed using the modified Heywood tooth stress formula are similar to those designed using the simpler Lewis formula for the example case studied here.

4. Design charts such as those shown here can be used for a single speed or over a range of speeds. For the sample gears in the study, a diametral pitch of 10.0 was found to provide compact gear set over the speed range considered.

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13. ABSTRACT ***(Maximum 200 words)***
This paper presents procedures for designing compact spur gear sets with the objective of minimizing the gear size. The allowable tooth stress and dynamic response are incorporated in the process to obtain a feasible design region. Various dynamic rating factors were investigated and evaluated. The constraints of contact stress limits and involute interference combined with the tooth bending strength provide the main criteria for this investigation. A three-dimensional design space involving the gear size, diametral pitch, and operating speed was developed to illustrate the optimal design of spur gear pairs. The study performed here indicates that as gears operate over a range of speeds, variations in the dynamic response change the required gear size in a trend that parallels the dynamic factor. The dynamic factors are strongly affected by the system natural frequencies. The peak values of the dynamic factor within the operating speed range significantly influence the optimal gear designs. The refined dynamic factor introduced in this study yields more compact designs than AGMA dynamic factors.

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